Warning: Some solutions are numerically wrong. Some fittings are not taken into account. Please, check them.

Hydrodynamics

Problem 1

Water is pressed up, from a vessel 'A', through a galvanized iron pipeline of diameter 1.6 cm to a vessel 'B' open to atmosphere. Water level in vessel 'A' may be taken constant. How much pressure must be provided over the water in vessel 'A' to lift up 0.724 m^3 liquid per hour to vessel 'B'? (ρ = 1000 kg/m^3, η = 1 mPas)

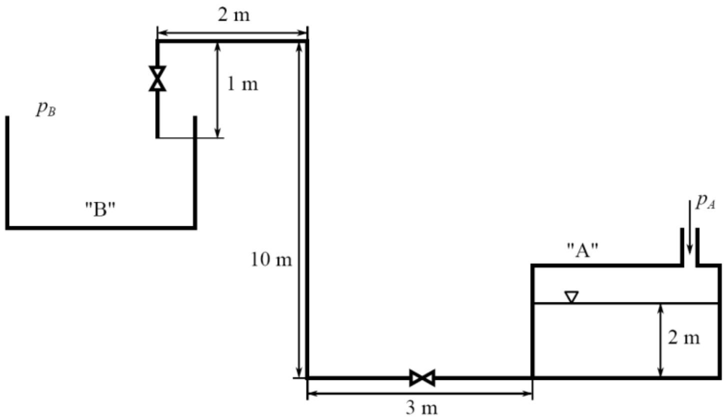

Figure 1, problem 1

Solution:

Apply Bernoulli equation between two points. Point 1 is taken as liquid level in vessel 'A', point 2 as the end of the pipeline.

Place of level h = 0 must also be assigned (arbitrarily). A good choice is the lowest point of the system because working with negative height is avoided this way.

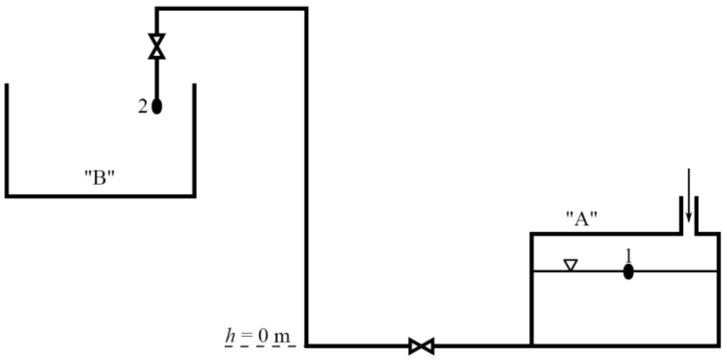

Figure 2. for Bernoulli equation and assigning necessary points in Problem 1

Bernoulli equation:

$$\frac{v_1^2 \cdot \rho}{2} + h_1 \cdot \rho \cdot g + p_1 = \frac{v_2^2 \cdot \rho}{2} + h_2 \cdot \rho \cdot g + p_2 + f \cdot \frac{L_{total}}{D_e} \cdot \frac{v_2^2 \cdot \rho}{2}$$

Pressure in point 1, i.e. p_1, is to be calculated.

Heights

According to the data given in the Figure, h_1 = 2 m and h_2 = 9 m.

Liquid velocities

v_1= 0 m/s because, as written in the Problem description, liquid level in vessel 'A' may be taken constant. (Note: This approximation is frequently applied if point 1 is taken at liquid level in a vessel of large cross section, and point 2 is in a pipeline. In this case, velocity of the level in the vessel is neglectable small compared to the velocity in the pipeline, according to the low of continuity.)

Velocity in point 2 can be calculated from volumetric flow rate.

$$\dot{V} = A \cdot v_2$$

$$v_2 = \frac{\dot{V}}{A} = \frac{\dot{V}}{\frac{D_{pipe}^2 \cdot \pi}{4}} = \frac{0.724\,\frac{m^3}{h}}{\frac{(0.016\,m)^2 \cdot \pi}{4}} \cdot \frac{1}{3600\,\frac{s}{h}} = 1\,\frac{m}{s}$$

Pressure

Pressure in point 2 is atmospheric, i.e. $p_2 = 10^5$ Pa.

Friction factor

First the Reynolds number must be calculated.

$$Re = \frac{D \cdot v \cdot \rho}{\eta} = \frac{0.016\,m \cdot 1\,\frac{m}{s} \cdot 1000\,\frac{kg}{m^3}}{10^{-3}\,Pas} = 1.6 \cdot 10^4$$

Relative roughness can be read from a *Relative roughness diagram of pipes*, based on the pipe's internal diameter and pipe material.

$$\left.\begin{matrix} D_{pipe} = 1.6\,cm \\ \text{galvanized iron} \end{matrix}\right\} \xrightarrow{\text{diagram}} \frac{\varepsilon}{D} = 0.01$$

Friction factor can then be read from the *Re – f diagram*.

$$\left.\begin{matrix} Re = 1.6 \cdot 10^4 \\ \frac{\varepsilon}{D} = 0.01 \end{matrix}\right\} \xrightarrow{\text{Re}-\text{f diagram}} f = 0.0415$$

Total pipe length

For determining friction loss, both losses of pipe and those of fittings must be taken into account. According to the figure, there are 2 open throughput vessels and 3 right angle elbows (elbow fittings 90°) in the pipeline. Total length is the sum of the geometric length and the equivalent length of the fittings. The latter one is the length of a straight pipe characterized with a resistance equal to those of the fittings.

$$L_{total} = L_{pipeline} + L_{equivalent}$$

Equivalent pipe length can be read from a nomogram A szerelvények egyenértékű csőhosszát az *Equivalent pipelength of fittings nomogram*: $L_{e,valve}$ = 5.5 m; $L_{e,elbow}$ = 0.32 m.

Total length is then

$$L_{total} = L_{pipe} + L_e = L_{pipe} + 2 \cdot L_{e,valve} + 3 \cdot L_{e,elbow} = 16\,m + 2 \cdot 5.5\,m + 3 \cdot 0.32\,m = 27.96\,m$$

Equivalent (hydraulic) pipe diameter

This equals the internal diameter because of applying circular tubes.

Pressure

Already just p_1 is unknown in Bernoulli equation.

$$h_1 \cdot \rho \cdot g + p_1 = \frac{v_2^2 \cdot \rho}{2} + h_2 \cdot \rho \cdot g + p_2 + f \cdot \frac{L_{total}}{D_e} \cdot \frac{v_2^2 \cdot \rho}{2}$$

$$p_1 = (h_2 - h_1) \cdot \rho \cdot g + p_2 + \left(f \cdot \frac{L_{total}}{D_e} + 1 \right) \cdot \frac{v_2^2 \cdot \rho}{2}$$

$$p_1 = (9\,m - 2\,m) \cdot 1000 \frac{kg}{m^3} \cdot 9.81 \frac{m}{s^2} + 10^5\,Pa + \left(0.0415 \cdot \frac{27.96}{0.016\,m} + 1 \right) \cdot \frac{\left(1 \frac{m}{s}\right)^2 \cdot 1000 \frac{kg}{m^3}}{2}$$

$$p_1 = 2.05 \cdot 10^5\,Pa$$

Problem 2

Liquid is transported from vessel 'A' to the open vessel 'B' through steel trade pipeline of 2 cm. Liquid density is 920 kg/m^3, viscosity is 0.8 mPas.

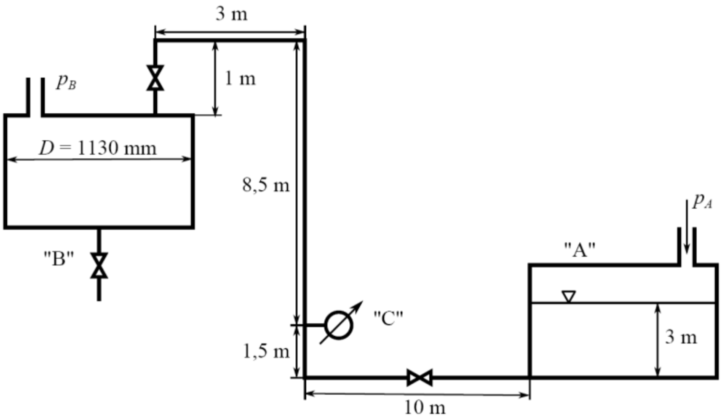

Figure 3, problem 2

a) In the time momment of opening the throughput valve, vessel 'B' is empty and the valve at the bottom of the vessel is closed. How long does it take the to reach a liquid level 1 m if 1.325 bar gauge is applied in vessel 'A'? Liquid level in vessel 'A' may be taken constant.

b) How much pressure is shown by manometer 'C' during the process of filling up vessel 'B'?

c) When liquid level in vessel 'B' reaches 1 m, the valve below the vessel is open. The liquid flows out through a pipe section of internal diameter 2 cm and lenght 1 m and a roughly worked hole (α = 0.8). What will be the liquid level in the vessel at steady state? Friction loss of the outlet pipe and in the vessel can be neglected.

d) Filling up vessel 'B' is stopped at steady state. How long does it take to decrease the liquid level in vessel 'B' to 10 cm?

Solution:

a) In the time momment of opening the throughput valve, vessel 'B' is empty and the valve at the bottom of the vessel is closed. How long does it take the liquid level to reach 1 m if 1.325 bar gauge is applied in vessel 'A'? Liquid level in vessel 'A' may be taken constant.

To solve problem a/, the volumetric flow rate and velocity of the liquid is to be calculated. zFor this aim, Bernoulli equation between two points is to be applied. Let point 1 be the liquid level in vessel 'A', and point 2 be the end of the pipeline.

Place of level h = 0 must also be assigned (arbitrarily). A good choice is the lowest point of the system because working with negative height is avoided this way.

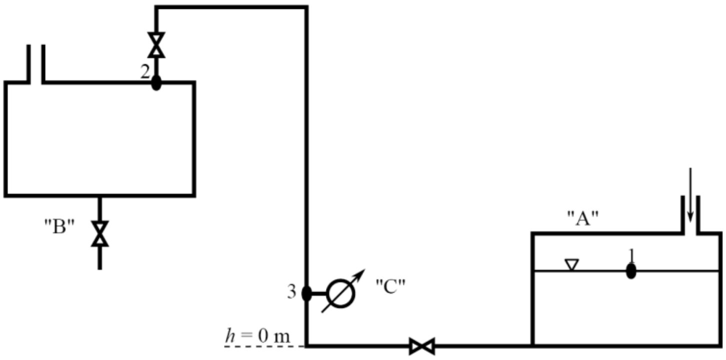

Figure 4, Points assignment for applying Bernoulli equation in Problem 2

Bernoulli equation

$$\frac{v_1^2 \cdot \rho}{2} + h_1 \cdot \rho \cdot g + p_1 = \frac{v_2^2 \cdot \rho}{2} + h_2 \cdot \rho \cdot g + p_2 + f \cdot \frac{L_{total}}{D_e} \cdot \frac{v_2^2 \cdot \rho}{2}$$

Velocity in point 2, i.e. v_2 is to be calculated.

Heights

According to the data given in the Figure, h_1 = 3 m and h_2 = 10 m.

Liquid velocity

v_1= 0 m/s because, as written in the Problem description, liquid level in vessel 'A' may be taken constant.

Pressure data

1.325 bar gauge pressure is applied in vessel 'A' according to the problem description, i.e. $p_1 = 2.325 \cdot 10^5$ Pa.

There is atmospheric pressure in point 2, i.e. $p_2 = 10^5$ Pa.

Total pipe length

According to the figure, there are 1 open throughput vessel and 2 right angle elbows (elbow fittings 90°) in the pipeline. Resistance of the manometer's junction may be neglected (because the liquid does not flow in the direction of the manometer).

Equivalent pipe length can be read from a nomogram A szerelvények egyenértékű csőhosszát az *Equivalent pipelength of fittings nomogram*: $L_{e,valve}$ = 6.5 m; $L_{e,elbow}$ = 0.4 m.

Total length is then

$$L_{total} = L_{pipe} + L_e = L_{pipe} + L_{e,valve} + 2 \cdot L_{e,elbow} = 24\,m + 6.5\,m + 2 \cdot 0.4\,m = 31.3\,m$$

Equivalent (hydraulic) pipe diameter

This equals the internal diameter because of applying circular tubes.

Friction factor

Friction factor is unknown. Its value depends on the also unknown liquid velocity. Such problems can be solved by either and iteration process or by applying Kármán's method.

Computation of liquid velocity by iteration

1. An initial friction factor is estimated using the *Re – f diagram*. A good choice (suggested) is the value read at such a large Reynolds number where the friction factor already does not depend on the Reynolds number.
2. Liquid velocity is calculated from the Bernoulli equation, using this friction factor.
3. A new friction factor is read from the *Re – f diagram*, using the new Reynolds number calculated with this liquid velocity, and using the relative roughness.
4. If the deviation of new friction factor from the previous one is smaller than a prescribed error then the result is accepted. Otherwise the iteration is continued with the new friction factor. The error margin should be determined according to the actual rquirements. Here we accept no more that 5% relative error.

$$
\begin{tikzcd}
\left.\begin{array}{l}\dfrac{\varepsilon}{D}\\ \text{high Re}\end{array}\right\} \arrow[r, "\text{Re-f diagram}"] & f \arrow[r, "\text{Bernoulli equation}"] & v \arrow[r, "\text{Re}, \frac{\varepsilon}{D}"] & f \arrow[d] \\
& {} \arrow[u] & & {} \arrow[ll]
\end{tikzcd}
$$

Figure 1.5. Determination of velocity by iteration

Relative roughness

Relative roughness is read from a *Relative roughness diagram of pipes*, based on the pipe's internal diameter and pipe material.

$$\left.\begin{array}{l} D_{pipe} = 2\,\text{cm} \\ \text{steel trade} \end{array}\right\} \xrightarrow{\text{Relative roughness diagram}} \frac{\varepsilon}{D} = 0.0025$$

Estimated friction factor

$$\left.\begin{array}{l} \text{high Re} \\ \dfrac{\varepsilon}{D} = 0.0025 \end{array}\right\} \xrightarrow{\text{Re}-\text{f diagram}} f_{estimated} = 0.025$$

Liquid velocity

From Bernoulli equation, with the estimated friction factor

$$h_1 \cdot \rho \cdot g + p_1 = h_2 \cdot \rho \cdot g + p_2 + \left(f \cdot \frac{L_{total}}{D_e} + 1 \right) \cdot \frac{v_2^2 \cdot \rho}{2}$$

$$v_2 = \sqrt{\frac{(h_1 - h_2) \cdot \rho \cdot g + p_1 - p_2}{\left(f_{estimated} \cdot \frac{L_{total}}{D_e} + 1 \right) \cdot \frac{\rho}{2}}}$$

$$v_2 = \sqrt{\frac{(3\,\text{m} - 10\,\text{m}) \cdot 920 \frac{\text{kg}}{\text{m}^3} \cdot 9.81 \frac{\text{m}}{\text{s}^2} + 2{,}325 \cdot 10^5\,\text{Pa} - 10^5\,\text{Pa}}{\left(0.025 \cdot \frac{31.3\,\text{m}}{0.02\,\text{m}} + 1 \right) \cdot \frac{920 \frac{\text{kg}}{\text{m}^3}}{2}}} = 1.94 \frac{\text{m}}{\text{s}}$$

Reynolds number

$$Re = \frac{D_e \cdot v_2 \cdot \rho}{\eta} = \frac{0{,}02\,m \cdot 1.94\,\frac{m}{s} \cdot 920\,\frac{kg}{m^3}}{0.8 \cdot 10^{-3}\,Pas} = 4.46 \cdot 10^4$$

New friction factor, From the *Re – f diagram*

$$\left.\begin{array}{l} Re = 4.46 \cdot 10^4 \\ \dfrac{\varepsilon}{D} = 0.0025 \end{array}\right\} \xrightarrow{\text{Re}-\text{f diagram}} f = 0.00285$$

The new friction factor is higher then the estimated one $f_{estimated} = 0.0025$ értéktől with 14%. This deviation is larger than the criterion 5%, thus the ieration in continued.

New liquid velocity

$$v'_2 = \sqrt{\frac{(h_1 - h_2) \cdot \rho \cdot g + p_1 - p_2}{\left(f \cdot \frac{L_{total}}{D_e} + 1\right) \cdot \frac{\rho}{2}}}$$

$$= \sqrt{\frac{(3\,m - 10\,m) \cdot 920\,\frac{kg}{m^3} \cdot 9.81\,\frac{m}{s^2} + 2.325 \cdot 10^5\,Pa - 10^5\,Pa}{\left(0.0285 \cdot \frac{31.3\,m}{0.02\,m} + 1\right) \cdot \frac{920\,\frac{kg}{m^3}}{2}}} = 1.82\,\frac{m}{s}$$

New Reynolds number

$$Re' = \frac{D_e \cdot v'_2 \cdot \rho}{\eta} = \frac{0{,}02\,m \cdot 1.82\,\frac{m}{s} \cdot 920\,\frac{kg}{m^3}}{0.8 \cdot 10^{-3}\,Pas} = 4.2 \cdot 10^4$$

New friction factor

$\left.\begin{array}{l} Re' = 4.2 \cdot 10^4 \\ \dfrac{\varepsilon}{D} = 0.0025 \end{array}\right\} \xrightarrow{\text{Re}-\text{f diagram}} f' = 0.0029$. Now the deviation from the previous one is only 3.6%; this is in the acceptable range therefore the results are accepted.

Volumetric flow rate

$$\dot{V}_2 = v'_2 \cdot A_{pipe} = v'_2 \cdot \frac{(Dpipe)^2 \cdot \pi}{4} = 1.82\,\frac{m}{s} \cdot \frac{(0.02\,m)^2 \cdot \pi}{4} = 5.72 \cdot 10^{-4}\,\frac{m^3}{s}$$

Volume of the liquid flown through

$$V = h_B \cdot A_{vessel} = h_B \cdot \frac{(D_{vessel})^2 \cdot \pi}{4} = 1m \cdot \frac{(1.13\,m)^2 \cdot \pi}{4} = 1m^3$$

Flow time till reaching the needed level

$$t = \frac{V}{\dot{V}} = \frac{1m^3}{5.72 \cdot 10^{-4}\,\frac{m^3}{s}} = 1749\,s = 29.15\,min$$

Kármán method

Kármán's method can be used to determine liquid velocity, based on pressure drop due to friction. However, if v_2 is unknown then pressure drop due to friction cannot be expressed from Bernoulli equation because of the member containing v_2.

Here we apply an approximation that the member containing v_2 is also included in pressure drop due to friction.

$$\Delta p_{friction} = f \cdot \frac{L_{total}}{D_e} \cdot \frac{v_2^2 \cdot \rho}{2}$$

$$\Delta p_{loss} = f \cdot \frac{L_{total}}{D_e} \cdot \frac{v_2^2 \cdot \rho}{2} + \frac{v_2^2 \cdot \rho}{2} = \left(f \cdot \frac{L_{total}}{D_e} + 1 \right) \cdot \frac{v_2^2 \cdot \rho}{2}$$

This approximation is applicable if $f \cdot \frac{L_{total}}{D_e} >> 1$. If, for example, $f \cdot \frac{L_{total}}{D_e} >> 20$ then the error is smaller thn 5%. Before applying Kármán's method, this error margin must be checked.

1. An initial friction factor is estimated using the *Re – f diagram*. A good choice (suggested) is the value read at such a large Reynolds number where the friction factor already does not depend on the Reynolds number.
2. If $\dfrac{f_{estimated} \cdot \frac{L_{total}}{D_e}}{1 + f_{estimated} \cdot \frac{L_{total}}{D_e}} > 0.95$ then the results will be in the range of 5%. (The error margin should be determined according to the actual rquirements. Here we accept no more that 5% relative error.)
3. If the above relation is not satisfied then the following pressure drop is used in the calculations:
$$\Delta p := \Delta p_{loss} \cdot \frac{f_{estimated} \cdot \frac{L_{total}}{D_e}}{1 + f_{estimated} \cdot \frac{L_{total}}{D_e}}$$
4. In this (latter) case one should check if the difference of the above Δp and $\Delta p_{friction}$ calculated from the new velocity is smaller then the error margin. If not then a $\Delta p_{friction}$ is calculated with the actually computed velocity, and the Kármán method is repeated (iterated) until acceptable values are resulted.
5. The value of $Re \cdot \sqrt{f}$ is calculated using the pressure difference (Δp_{loss} or the corrected Δp).
$$Re \cdot \sqrt{f} = \frac{D_e \cdot \rho}{\eta} \cdot \sqrt{\frac{2 \cdot D_e \cdot \Delta p}{L_{total} \cdot \rho}}$$
6. Once $Re \cdot \sqrt{f}$ and the relative roughness are known, $\frac{1}{\sqrt{f}}$ is read from the $Re \cdot \sqrt{f} - \frac{1}{\sqrt{f}}$ *diagram* or calculated with the Colebrook formula:
$$\frac{1}{\sqrt{f}} = -2 \cdot \log\left(\frac{2.51}{Re \cdot \sqrt{f}} + \frac{1}{3.72} \cdot \frac{\varepsilon}{D} \right)$$
7. Liquid velocity is then calculated.
$$v = \frac{1}{\sqrt{f}} \cdot \sqrt{2 \cdot \frac{\Delta p \cdot D_e}{L_{total} \cdot \rho}}$$

Estimated friction factor

$$\left.\begin{array}{l}\text{high Re}\\ \dfrac{\varepsilon}{D}=0.0025\end{array}\right\}\xrightarrow{\text{Re}-\text{f diagram}} f_{estimated}=0{,}025$$

Preliminary check of the Kármán method

$$\frac{f_{estimated}\cdot\dfrac{L_{total}}{D}}{1+f_{estimated}\cdot\dfrac{L_{total}}{D}}=\frac{0.025\cdot\dfrac{31.3\,m}{0.02\,m}}{1+0.025\cdot\dfrac{31.3\,m}{0.02\,m}}=0.975$$

This is larger than 0.95, therefore the error is smaller than 5%.

Pressure loss

$$h_1\cdot\rho\cdot g+p_1=\frac{v_2^2\cdot\rho}{2}+h_2\cdot\rho\cdot g+p_2+f\cdot\frac{L_{total}}{D_e}\cdot\frac{v_2^2\cdot\rho}{2}$$

$$\Delta p_{loss}=f\cdot\frac{L_{total}}{D_e}\cdot\frac{v_2^2\cdot\rho}{2}+\frac{v_2^2\cdot\rho}{2}=(h_1-h_2)\cdot\rho\cdot g+p_1-p_2$$

$$\Delta p_{loss}=(3\,m-10\,m)\cdot 920\frac{kg}{m^3}\cdot 9.81\frac{m}{s^2}+2.325\cdot 10^5\,Pa-10^5\,Pa=6.93\cdot 10^4\,Pa$$

Determination of $\dfrac{1}{\sqrt{f}}$

$$Re\cdot\sqrt{f}=\frac{D_e\cdot\rho}{\eta}\cdot\sqrt{\frac{2\cdot D_e\cdot\Delta p}{L_{total}\cdot\rho}}=\frac{0.02\,m\cdot 920\dfrac{kg}{m^3}}{0.8\cdot 10^{-3}\,Pas}\cdot\sqrt{\frac{2\cdot 0.02\,m\cdot 6.93\cdot 10^4\,Pa}{31.3\,m\cdot 920\dfrac{kg}{m^3}}}=7137$$

Based on the $Re\cdot\sqrt{f}-\dfrac{1}{\sqrt{f}}$ *diagram*:

$$\left.\begin{array}{l}Re\cdot\sqrt{f}=7.1\cdot 10^3\\ \dfrac{\varepsilon}{D}=0.0025\end{array}\right\}\xrightarrow{Re\cdot\sqrt{f}-\frac{1}{\sqrt{f}}\text{ diagram}}\frac{1}{\sqrt{f}}=5.8$$

Based on the Colebrook formula:

$$\frac{1}{\sqrt{f}}=-2\cdot\log\left(\frac{2.51}{Re\cdot\sqrt{f}}+\frac{1}{3.72}\cdot\frac{\varepsilon}{D}\right)=-2\cdot\log\left(\frac{2.51}{7137}+\frac{1}{3.72}\cdot 0.0025\right)=5.98$$

Liquid velocity

$$v_2=\frac{1}{\sqrt{f}}\cdot\sqrt{2\cdot\frac{\Delta p\cdot D_e}{L_{total}\cdot\rho}}=5.98\cdot\sqrt{2\cdot\frac{6.93\cdot 10^4\,Pa\cdot 0.02\,m}{31.3\,m\cdot 920\dfrac{kg}{m^3}}}=1.86\frac{m}{s}$$

Volumetric flow rate

$$\dot{V}_2=v_2\cdot A_{tube}=v_2\cdot\frac{(D_{tube})^2\cdot\pi}{4}=1.86\frac{m}{s}\cdot\frac{(0.02\,m)^2\cdot\pi}{4}=5.83\cdot 10^{-4}\,\frac{m^3}{s}$$

Time

$$t=\frac{V}{\dot{V}}=\frac{1\,m^3}{5.83\cdot 10^{-4}\,\dfrac{m^3}{s}}=1716\,s=28.6\,min$$

b) How much pressure is shown by manometer 'C' during the process of filling up vessel 'B'?

For answering this question, Bernoulli equation is applied between point 1 and point 3, the place of the manometer junction. (Note: the same result is obtained applying it between points 2 and 3.

Bernoulli equation

$$\frac{v_1^2 \cdot \rho}{2} + h_1 \cdot \rho \cdot g + p_1 = \frac{v_3^2 \cdot \rho}{2} + h_3 \cdot \rho \cdot g + p_3 + f \cdot \frac{L'_{total}}{D_e} \cdot \frac{v_3^2 \cdot \rho}{2}$$

Pressure in point 3, i.e. p_3 is aimed.

Height

According to the data in the figure, h_3 = 1.5 m.

Liquid velocity

Because of the continuity and and equal diameters, it is the same as determined in problem a/: $v_3 = v_2$ = 1.86 m/s.

Total pipe length

There are 1 valve and 1 db elbow between points 1 and 3.

$$L'_{total} = L'_{pipe} + L'_e = L'_{pipe} + L_{e,valve} + L_{e,elbow} = 11.5\,m + 6.5\,m + 0.4\,m = 18.4\,m$$

Friction factor

Friction factor is computed in problem a/ as f = 0,029. (It can also be computed from the value $\frac{1}{\sqrt{f}}$ = 5.98 obtained with Kármán's method, as f = 0,028.)

Pressure

$$h_1 \cdot \rho \cdot g + p_1 = \frac{v_3^2 \cdot \rho}{2} + h_3 \cdot \rho \cdot g + p_3 + f \cdot \frac{L'_{total}}{D_e} \cdot \frac{v_3^2 \cdot \rho}{2}$$

$$p_3 = (h_1 - h_3) \cdot \rho \cdot g + p_1 - \left(f \cdot \frac{L'_{total}}{D_e} + 1 \right) \cdot \frac{v_3^2 \cdot \rho}{2}$$

$$p_3 = (3\,m - 1.5\,m) \cdot 920 \frac{kg}{m^3} \cdot 9.81 \frac{m}{s^2} + 2.325 \cdot 10^5\,Pa - \left(0.028 \cdot \frac{18.4\,m}{0.02\,m} + 1 \right) \cdot \frac{\left(1.86 \frac{m}{s}\right)^2 \cdot 920 \frac{kg}{m^3}}{2}$$

$$p_3 = 2.03 \cdot 10^5\,Pa$$

The manometer shows gauge pressure, i.e. $p'_3 = 1.03 \cdot 10^5$ Pa.

c) When liquid level in vessel 'B' reaches 1 m, the valve below the vessel is open. The liquid flows out through a pipe section of internal diameter 2 cm and lenght 1 m and a roughly worked hole (α = 0.8). What will be the liquid level in the vessel at steady state? Friction loss of the outlet pipe and in the vessel can be neglected.

Inflow to and outflow from vessel 'B' are equal in steady state. Volumetric flow rate can be calculated from liquid velocity determined in problem a/, and then outflow velocity can be calculated. Since friction loss in the outflow pipe is neglected, the formula valid for free outflow can be applied to determine which level causes the calculated outflow velocity.

Volumetric flow rates

$$\dot{V}_2 = v_2 \cdot A_{tube} = v_2 \cdot \frac{(D_{tube})^2 \cdot \pi}{4} = 1.86\,\frac{m}{s} \cdot \frac{(0{,}02\,m)^2 \cdot \pi}{4} = 5.84 \cdot 10^{-4}\,\frac{m^3}{s}$$

$$\dot{V}_{out} = \dot{V}_2 = 5.84 \cdot 10^{-4}\,\frac{m^3}{s}$$

Outflow velocity

One should take into account that, because of rough elaboration, only 80 % of the outflow cross section is useful.

$$v_{out} = \frac{\dot{V}_{out}}{A_{out}} = \frac{\dot{V}_{out}}{\alpha \cdot \frac{(D_{out})^2 \cdot \pi}{4}} = \frac{5.84 \cdot 10^{-4}\,\frac{m^3}{s}}{0.8 \cdot \frac{(0.02\,m)^2 \cdot \pi}{4}} = 2.325\,\frac{m}{s}$$

Liquid level

$$v_{out} = \sqrt{2 \cdot g \cdot h_B}$$

$$h_B = \frac{v_{out}^2}{2 \cdot g} = \frac{\left(2.325\,\frac{m}{s}\right)^2}{2 \cdot 9.81\,\frac{m}{s^2}} = 0.28\,m$$

Thus the liquid level in vessel 'B' at steady state is 28 cm. (Note: This steady state level is independent of the level at the time moment of opening the outflow valve.)

d) Filling up vessel 'B' is stopped at steady state. How long does it take to decrease the liquid level in vessel 'B' to 10 cm?

Since friction loss in the outflow pipe is neglected, the formula valid for free outflow can be applied. For applying this formula, the basis level h = 0 m should be set to the point where the level would decrease in infinite time. This is the lower end of the outflow pipe in our case. The length of the pipe is given as 1 m in description of problem c/.

Liquid levels

As is calculated in problem c/, the liquid level in vessel 'B' at steady state is 28 cm. This is from where the level will decrease to 10 cm. These heights related to the new basis h = 0 m are: h_0 = 1.28 m and h_1 = 1.1 m.

Time needed for outflow

$$t = \frac{2 \cdot \frac{(D_{vessel})^2 \cdot \pi}{4}}{\alpha \cdot \frac{(D_{out})^2 \cdot \pi}{4} \cdot \sqrt{2 \cdot g}} \cdot \left(\sqrt{h_0} - \sqrt{h_1}\right) = \frac{2 \cdot \frac{(1.13\,m)^2 \cdot \pi}{4}}{0.8 \cdot \frac{(0.02\,m)^2 \cdot \pi}{4} \cdot \sqrt{2 \cdot 9.81\,\frac{m}{s^2}}} \cdot \left(\sqrt{1.28\,m} - \sqrt{1.1\,m}\right)$$

$$t = 148.8\,s \approx 2.5\,min$$